A Rheological Study of Laminar–Turbulent Transition in Drag Reducing Polymeric Solutions

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Synopsis

Presented herein are the studies of laminar–turbulent transition in micropolar and power law fluids flowing in a circular pipe. For some parametric values of micoropolar and power law fluids, both depict the drag reducing properties. The parametric values of these representations have been obtained from the experimental results of Mc-Comb. It has been observed that, in both the cases, as the drag-reducing property in the solution increases, the first transition point moves towards the walls of the pipe. It is also observed that the onset of early turbulence phenomenon occurs for the drag-reducing polymeric solutions.

INTRODUCTION

From the basic concept of the laminar–turbulence transition, it is pictured that the process of transition occurs in a few steps; the formation of 2-dimensional waves, 3-dimensional waves turbulent spot, and its propogation to the entire field of flow. In a pipe line design, for which one needs a means of ascertaining whether the flow will be laminar or turbulent, the Reynolds number is the criterion for the Newtonian fluids, but distinct regions of laminar–turbulent transition are also observed for other kind of fluids. Shirto et al.[1] investigated the process of laminar–turbulent transition of fluid flow in a circular pipe in detail and measured the velocity of fluctuations and compared the experimental values of critical position with the theoretical one. The problem of laminar–turbulent transition in drag-reducing polymeric solutions becomes more important as White and McElogot[2] have reported that the drag reducing polymeric solutions delay the turbulence, but Hansen and Little[3] are in favor of the early turbulence. A detailed experimental work of Jones et al.[4] is also in favor of the early turbulence.

In recent past much attention has been paid to study the laminar–turbulent transition in certain non-Newtonian fluids. In almost all the work appearing to date in reference to the polymer suspensions, the classical Navier–Stokes equations have been introduced. The inadequacy of the classical continuum approach to describe the mechanics of complex fluids such as liquid crystals and polymeric suspensions, etc., has led to the development of microcontinua; one such theory on microcontinua is that of micropolar fluids.[5]

In this paper we have presented a study of laminar–turbulent transition phenomenon for drag-reducing polymeric solutions by considering power law fluid (in the first part) and micropolar fluid (in the second part) representation for the drag-reducing polymeric solution, in a circular pipe for

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some parametric values of power law fluid and micropolar fluid. The parametric values of power laws representation have been reported by Tandon and Kulshreshtha[6] using the experimental results of McComb.[7] The results of this paper may be valuable in explaining the drag-reducing phenomenon which still needs a satisfactory rigor.

ANALYSIS

For Power Law Fluid Representation

From the Navier–Stokes equation for incompressible power law fluid, we obtain the equation of kinetic energy

$$\overline{U}\left(\overline{\rho}\,\frac{D\overline{U}}{Dt}\right) = \overline{\rho}\,(\overline{U}\cdot\overline{F})\text{ge} - (\overline{U}\cdot\nabla p)\text{ge} + \overline{U}(\nabla T) \tag{1}$$

where

$$T = m\left(\frac{1}{2}\Delta:\Delta\right)^{n-1/2}\Delta \tag{2}$$

Here Δ is the rate of deformation tensor, $\overline{U}$ is the velocity vector, $\overline{F}$ is the external force, $\overline{\rho}$ is the density, P is the pressure, t is the time, T is the stress tensor, and ge is the gravitational conversion factor. Throughout the analysis the external force $\overline{F}$ has been neglected.

The motion of the fluid is analyzed into the mean motion and superposed turbulent fluctuations. The velocity components and pressure of the total motion can be expressed in the component form as

$$u_i = \overline{u}_i + u_i', \quad i = r, \theta, \text{ or } z, \quad p = \overline{p} + p' \tag{3}$$

For the steady motion in circular pipes we introduce the velocity components as

$$u_r = u_r', \quad u_\theta = u_\theta', \quad u_z = \overline{u}_z + u_z' \tag{4}$$

As the disturbance is introduced at only one position, the corresponding differential terms of the velocity fluctuations with respect to the position are neglected and, further, as the fluctuations are very small. The square of the velocity fluctuations are also neglected. Thus, the kinetic energy equations may be written for axisymmetric flow of power law fluid in a circular pipe as

$$\begin{aligned}\overline{\rho}\,\frac{D}{Dt}\left(u_r'^2 + u_\theta'^2 + u_z'^2\right) + \overline{\rho}\,\frac{D}{Dt}(u_z'\overline{u}_z) \\ = -\,\text{ge}\left[u_r'\frac{\partial p'}{\partial r} + u_z'\frac{\partial}{\partial z}(\overline{p} + p') + \overline{u}_z\frac{\partial u'}{\partial z}\right] \\ + \phi - \psi\end{aligned} \tag{5}$$

where

$$\phi = mu'_z\left[\frac{\partial}{\partial r}\left(\left|\frac{\partial \overline{u}_z}{\partial r}\right|^{n-1}\frac{\partial \overline{u}_z}{\partial_r}\right) + \frac{1}{r}\left|\frac{\partial \overline{u}_z}{\partial r}\right|^{n-1}\right. \tag{6}$$

and

$$\psi = \overline{\rho} u'_r \overline{u}_z \frac{\partial \overline{u}_z}{\partial r} \tag{7}$$

The terms on the left-hand side of eq. (5) represent the rate of increase in the surplus kinetic energy per unit volume by the velocity fluctuations, ψ represents the rate of additional energy supplied from the base flow, and ϕ represents the rate of dissipation of the velocity fluctuation energy. If $\phi \geq \psi$, the disturbances may be damped out, and the flow remains laminar; if $\phi < \psi$, the disturbances will grow up, and the laminar flow will be unstable. Now we define the stability index

$$I_s = \psi/\phi \tag{8}$$

For the power law fluid, introducing.

$$\overline{u}_z = \langle u_z \rangle \frac{3n+1}{n+1}\left(1 - \rho\,\frac{(n+1)}{n}\right) \tag{9}$$

we get

$$I_s = \left[\rho/2n\left\{<u_z>\frac{3n+1}{n+1}\right\}^{2-n}\left\{\left(\frac{n+1}{n}\right)\right\}^{1-n}\right]\left[\rho 1/n\left\{1-\rho\,\frac{n+1}{n}\right\}\right] \tag{10}$$

where $\rho = r/R$ and $\langle u_z \rangle$ is the average velocity. The first disturbance in the motion of the fluid grows at the point where the stability index is maximum. The stability index is maximum at

$$dI_s/d\rho = 0 \tag{11}$$

i.e., at

$$\rho = \left(\frac{1}{n+2}\right)^{n/(n+1)} \tag{12}$$

For Micropolar Fluid Representation

The equation of kinetic energy for the micropolar fluid is

$$\rho \overline{U} \frac{D\overline{U}}{Dt} = (\lambda_a + 2\mu_a + k_a)\, \overline{U}\, \overline{\nabla}(\overline{\nabla}\, \overline{U}) - (\mu_a + k_a)\, \overline{U}\, (\overline{\nabla} \times \overline{\nabla} \times \overline{U}) + k_a \overline{U}(\overline{\nabla} \times \overline{W}) - \text{ge}\, \overline{U}\, \overline{\nabla} p + (\overline{U}\, \overline{F}) \quad (13)$$

where λ_a, μ_a, and k_a are parameters for the micropolar fluid and $\overline{W}$ is the gyration vector.

Following the same process and introducing the same assumptions as taken in the previous subsection in addition the components of the gyration are written as

$$W_r = W'_r,\; W_\theta = \overline{W}_\theta + W'_\theta \text{ and } W_z = W'_z \quad (14)$$

Finally, we obtain a kinetic energy equation similar to eq. (5), where

$$\phi = (\mu_a + k_a) u'_z \left(\frac{\partial^2}{\partial r^2} \overline{u}_z + \frac{1}{r} \frac{\partial}{\partial r} \overline{u}_z \right) + k_a u'_z \left[\frac{\partial}{\partial r} (\overline{W}_\theta) + \frac{1}{r} \overline{W}_\theta \right] \quad (15)$$

when $\lambda < 3.75$

For the micropolar fluid introducing

$$\overline{u}_z = 2\langle u_z \rangle (1 - \rho^2) \quad (16)$$

$$\overline{W}_\theta = 2\langle u_z \rangle \lambda^2 \rho (1 - \rho^2) / R(8 + \lambda^2 - 4N) \quad (17)$$

where

$$\lambda = \left(\frac{2\mu_a + k_a}{\mu_a + k_a} \cdot \frac{k_a}{\gamma_a} \right)^{1/2} R \quad (18)$$

and

$$N = k_a / (\mu_a + k_a) \quad (19)$$

We get the stability index

$$I_s = \frac{R\rho \langle u_z \rangle}{\mu_a + k_a} \frac{\rho (1 - \rho^2)}{1 + [N\lambda^2 (1 - 2\rho^2) / 2(8 + \lambda^2 - 4N)]} \quad (20)$$

The stability indez will be maximum at

$$\rho^2 = \frac{1}{4}\{(3N_1 + 1)[(3N_1+1)^2-8(N_1 + 1)]^{1/2}\} \tag{21}$$

where

$$N_1 = \frac{2(8+\lambda^2 - 4N)}{N\lambda^2} \tag{22}$$

CALCULATION OF THE CRITICAL VELOCITY

The maximum value of the stability index remains constant for all the fluids.[8] Therefore, the critical velocity at which the first transition point appears, for the power law fluid is

$$\langle U_{zp}\rangle = \frac{1}{3n+1}\left[\frac{4}{3^{3/2}}\frac{\rho_1}{\rho_1}\frac{m}{\mu}\, n^{1-n}\,(n+2)^{(n+2)/(n+1)}\right]^{1/2-n)} \times R^{(1-n)/(2-n)}\langle u_{zw}\rangle^{1/(2-n)} \tag{23}$$

and for the micropolar fluid is

$$\langle u_{zm}\rangle = \langle u_{zw}\rangle \frac{\rho_1}{\rho_2}\frac{\mu_a + k_a}{3^{3/2}\,\mu}\frac{X}{(1-\rho^2)} \tag{24}$$

where

$$X = 1 + \{N\lambda^2(1-2\rho^2)/2\,(8+\lambda^2 - 4N)\} \tag{25}$$

and $\langle u_{zw}\rangle$ is the critical velocity where the first disturbance grows in the water.

RESULTS AND DISCUSSION

Geometric Position of the First Critical Point

The formulation of the turbulent spot is a local phenomenon, and it has been observed that its formation concides with the point at which the stability index is maximum. Though the development of instability depends upon the local conditions but the critical radius does not remains uneffected by velocity profiles and rheological properties of the .luid.

Figure 1 depicts the variation of the geometric position of the first transition point with N (a nondimensional parameter of micropolar fluid). We observe that, as the value of N increases, the first transition point moves

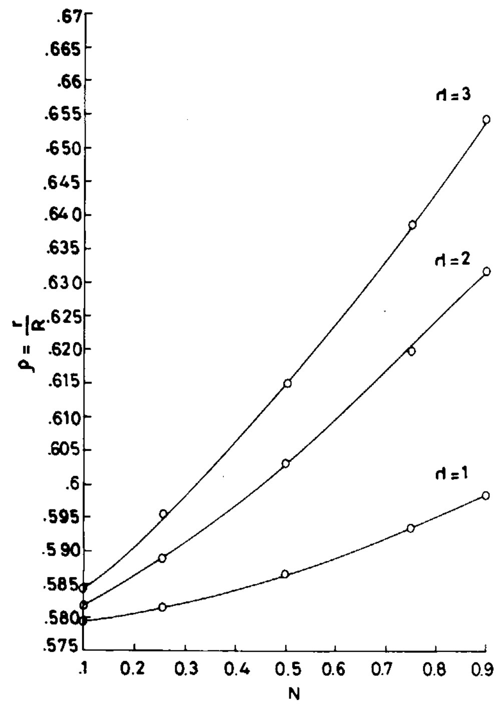

Fig. 1. Variation of critical point ($r/R=\rho$) with N for different values of λ.

towards the walls of the pipe and this fact is similar to that observed for the power law fluid for decreasing value of n (a parameter of power law fluid), shown in Figure 2 for pseudoplastic fluids. Therefore, we can say that as the value of N increases in the micropolar fluid, the drag-reducing property of the fluid increases. Also as the parameter λ increases, the rate of shifting of the first transition point towards the walls increases. This fact also shows that the increasing values of parameter λ are favorable for drag-reduction phenomenon similar to the pseudoplasticity of the polymer suspension as obtained by us[6] for the power law fluid from the data obtained for the experimental results of McComb.[7]

Early Transition

Figure 3 describes the variation of critical velocity at which the transition occurs, with the pseudoplasticity of the power law fluid taking $\rho_1 = \rho_2$, $\mu = 0.01$, and m $= 0.012$. We concluded that, as the pseudoplasticity of the fluid increases, the first disturbance appears earlier. Similar results have been reported by Hansen and Little[3] and Jones et al.[4] Mishra and Tripathi[8] have observed that the pseudoplasticity delays the laminar–turbulent transition which might be the effect of higher viscosity due to the addition of greater amount of polymer to the solvent.

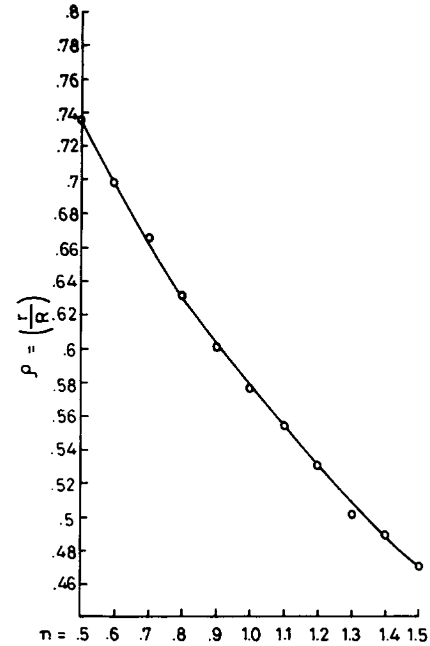

Fig. 2. Variation of critical point (r/R) with flow behavior index (n), in a circular pipe.

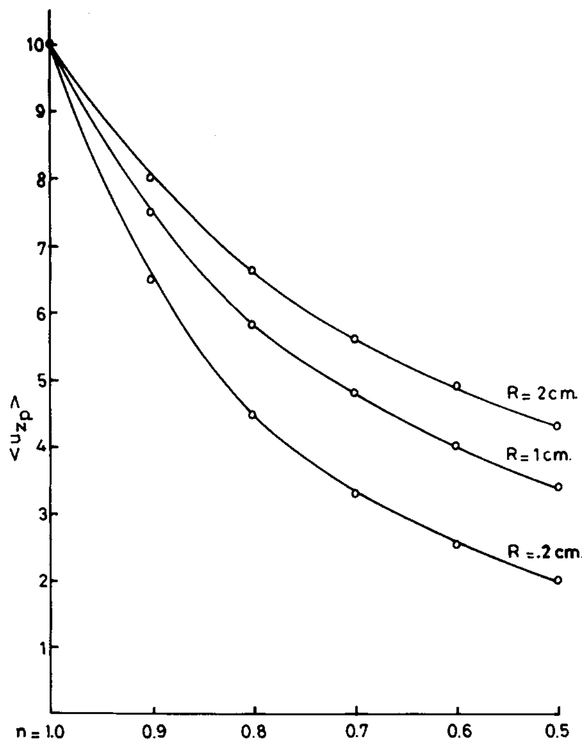

Fig. 3. Variation of critical velocity with pseudoplastisity where $\langle u_{zw} \rangle$ = 10 cm/sec.

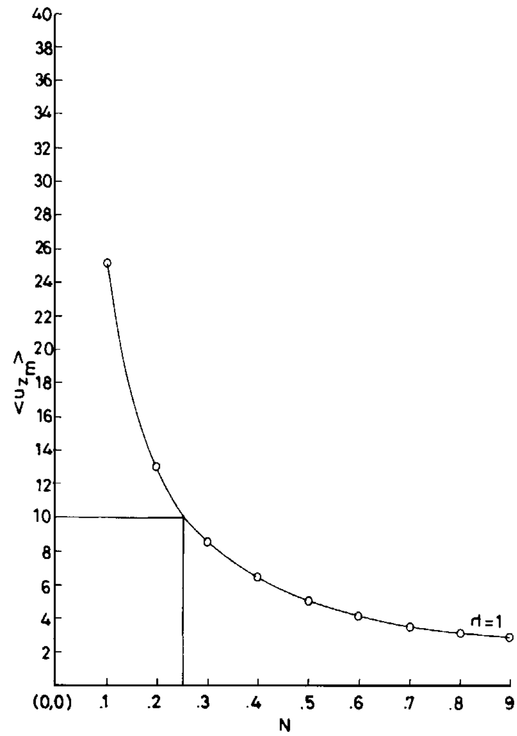

Fig. 4. Variation of the critical velocity $\langle u_{z_2} \rangle$ with N for different values of N where $K = 5 \times 10^{-3} \langle u_z \rangle = 10$ and $\lambda = 1$.

Figure 4 describes the variation of critical velocity at which the first transition occurs, with the parameter N of the micropolar fluid taking $\rho_1 = \rho_2$ and $K_\alpha = 5 \times 10^{-3}$, $\mu = 1 \times 10^{-2}$. From this figure, we observe that as the parameter N increases the critical velocity decreases i.e. the transition occurs earlier. This result is quantitatively similar to that of Harro Kuemmever[9] between two parallel walls.

From these results, we conclude that the onset of early turbulence phenomenon occurs in dilute solutions of the drag-reducing agents.

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